MATHEMATICS SYLLABUS D

Paper 4024/11
Paper 11

Key messages

All working should be shown and answers clearly written in the appropriate answer space.

This is a non-calculator paper and accuracy in basic number work is essential. A fluency in computational skills is beneficial.

Candidates are encouraged to take notice of instructions given within a question, for example **Question 3(b)** requires the answer to be in its simplest form, **Question 14(a)** asks for the next term only, and **Question 23(a)** requires the answer to be given in standard form.

General comments

The performance on this paper was generally good with many candidates demonstrating sound understanding of most aspects of the syllabus. The majority of candidates attempted all the questions.

Incorrect cancelling was often seen in **Questions 7**, **10** and **23**.

The questions that candidates found most difficult were **Questions 17**, **22** and **25** and parts of **Questions 6**, **20** and **23**. Those most commonly omitted were parts of **Questions 11**, **16**, **20** and **23**.

Comments on specific questions

Question 1

(a) Most candidates answered this correctly. Not all multiplied by 10 or 100 correctly and a common incorrect answer was $74.6 \times 10 = 740.6$. Some made an arithmetic error in the subtraction.

(b) Most candidates answered this correctly. Common incorrect answers were 8 and 15.

Question 2

(a) Most candidates answered this correctly. Some candidates stated that 15 was a factor of 70.

(b) Most candidates were able to correctly identify the cube number.

(c) Many candidates answered this part correctly. A few candidates stated that $\sqrt{25}$ was irrational and others gave 11 as their answer.

Question 3

(a) Most candidates were able to add the two fractions together correctly. A few made arithmetic errors and the incorrect answer 5/12 was occasionally seen.

(b) Most candidates answered this part correctly. Some did not give their answer in its simplest form. A few candidates attempted a form of cross multiplication, obtaining an answer of 18/22.

Question 4

(a) Most candidates answered this part correctly. A few gave the answer 3, from 5 − 2.

(b) Many candidates found this part more challenging than **(a)**. Many obtained the correct answer while −6.7 and 24.3 were among the incorrect answers seen. Some candidates omitted the minus sign from their answer.

Question 5

Most candidates earned a mark for finding $C\hat{D}A = 58°$ and the majority went on to obtain the correct answer. Some arithmetic errors were seen. Some candidates gave answers such as x = 39° (from 122° − 83°) and 52° (from 135° − 83°).

Question 6

(a) Many correct answers were seen. Some candidates made slips in their division resulting in the incorrect factors. Occasionally candidates left their answer as $2^2 \times 77$.

(b) Many candidates found this part more challenging than **(a)** and 2 and 11 were common incorrect answers.

Question 7

Many candidates answered this correctly. Some candidates used the wrong formula for the area of a trapezium, for example $\frac{1}{2}(7+10)+h$ and $\frac{1}{2}(7\times10)+h$ were both seen.

Some candidates cancelled incorrectly. An example typically seen was $\frac{1}{2}(7+10)h$ cancelled to $(7+5)h$ cancelled to $12h$. Some candidates made slips when calculating the value of $\frac{1}{2}\times17$.

Question 8

(a) Most candidates answered this part correctly. A few candidates changed the question into an equation, attempting to solve $6x + 15 = 2x + 8$.

(b) Some candidates expanded $(x - 5)^2$ as $(x - 5)(x + 5)$ obtaining the answer $x^2 - 25$. A few, having correctly expanded the expression to $x^2 - 5x - 5x + 25$, went on to give the answer $x^2 + 25$.

Question 9

(a) Many candidates answered this part correctly.

(b) Many candidates answered this part correctly.

(c) Many candidates were less successful in this part and '=' was a common incorrect answer.

Question 10

Many candidates answered this correctly having followed the instruction to write each number correct to one significant figure. A few wrote the numbers as 4, 2 and 5 giving the answer 2/5, and some wrote 362.5 as 360.

Some incorrect cancelling was seen, for example $\frac{400-200}{50}$ cancelling to $\frac{80-20}{5}$.

Question 11

(a) Many correct answers were seen. A few gave the answer $\frac{1}{97}$. Candidates are expected to work with probabilities in the form of a fraction, decimal or percentage.

(b) (i) Some candidates found the total number of people and then found the number preferring plan *A.* A few gave the answer 156 from 52×3.

(ii) This part was answered well by most candidates.

Question 12

Many candidates realised that they needed to divide 15 by 0.075 but some had difficulty in doing this correctly. It was common for candidates to use the wrong method and to attempt 15×0.075.

Question 13

Many candidates gave the correct answer to this part. Several different methods were seen. Some candidates obtained two equations involving *x* and *y* and solved them simultaneously. Others divided 112 in the ratio 5 : 11 but sometimes had difficulty dividing 112 by 16.

Question 14

(a) Most candidates answered this part correctly. A few gave more than the one (subsequent) term required.

(b) (i) Many candidates answered this correctly. Some wrote that the relationship between terms was subtract 3 and gave the incorrect answer −7, while a few gave the 5th term of the sequence (116). Some candidates attempted to square −4 but made an error in their calculation and gave the answer −21.

(ii) Many candidates gave the two values correctly. Some gave just 2 omitting the −2. A common incorrect answer was 4 or −4.

Question 15

(a) Many candidates gave the correct time. A variety of incorrect answers were seen including 13 04 (from 15 10 − 2 14), 13 56, 11 56 and 1246.

(b) Most candidates began their response with either an attempt at calculating the speed (15÷12) or converting the time to hours (12÷60). Some were unable to use these correctly to obtain the answer, making slips in calculating 15 ÷ 0.2, or stating the answer 1.25, or multiplying 12/60 by 15.

(c) Many candidates found this part challenging. Common incorrect answers were 232° (from 360° − 128°) and 52° (from 180° − 128°). It should be noted that the syllabus specifies three-figure bearings.

Question 16

Some candidates found this question challenging.

(a) Many candidates found this part difficult and gave the wrong transformation. Others realised that the transformation is a reflection but had trouble identifying the mirror line and gave the mirror line as $y = -1$ or $x = -2$.

(b) Again many found this part challenging. Some were able to identify that the transformation is a rotation but were not able to state the angle of rotation and/or the centre of rotation. The centre of rotation should be stated as (0,1) and not $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and the direction of rotation needs to be stated as clockwise (or equivalent).

(c) Some candidates omitted this part and few candidates were able to draw the correct triangle *D*. Some candidates drew an enlargement of triangle *A* but used the wrong centre of enlargement, most commonly (0,0).

Question 17

Most candidates found this question challenging.

Candidates commonly stated equal angles and lengths but did not give reasons. Some candidates identified lines with equal length and angles of the same size. Some candidates stated that three equal angles was a condition for congruency.

Question 18

This question was answered very well by the majority of candidates. A few made arithmetic errors but many completely correct solutions were seen.

Question 19

Many candidates did well on this question. Some candidates did not insert a constant into their equation and worked with $y = (x - 1)^2$, reaching the answer 25.

Many, having got as far as $18 = k(4 - 1)^2$ went on to state that $18 = k(4^2 - 1^2)$ giving $k = 18/15$. Others followed a similar approach, working instead with $(6 - 1)^2$.

Question 20

(a) (i) Most candidates drew the line $y = 2$ correctly although a few drew $x = 2$.

(ii) Some candidates drew a line that was too short to cut both *x* and *y* axes.

(b) Some candidates were able to shade the correct region and others gained credit for shading a region which satisfied three of the given inequalities.

Question 21

(a) The expression was correctly factorised by most candidates. Those who did not complete the factorisation correctly usually earned a mark for some correct partial factorisation, for example $3c(x - 2y) + 2(x - 2y)$ leading to $(3c + 2)(x - 2y)$ or $x(3c + 2b) - 2(3c + 2b)$ leading to $(x - 2)(3c + 2b)$.

(b) Many candidates were able to factorise the expression correctly. Some were able to factorise so as to get two of the terms correct, for example $(6x + 5)(x - 2)$ or $(3x + 10)(2x - 1)$. Others attempted to complete the square or use the formula as if solving quadratic equation.

Question 22

Many good answers were seen. A few candidates did not seem to realise the significance of 'correct to the nearest hundred kilograms' and 'correct to the nearest ten kilograms' and added 2400 and 1460. Some candidates, having obtained a total of 3860, then attempted to find a lower bound for this and gave the answer 3855 or 3859.5. Others found the wrong lower bounds for the individual masses, for example 2300 and 1450 and added them together giving the answer 3750.

Question 23

(a) (i) This part proved challenging to some candidates. The most common error was either to forget to square 4×10^2 or to square it incorrectly; an example of a common incorrect answer seen was 800. A few, having got to the stage of $\frac{16 \times 10^4 + 6 \times 10^4}{2 \times 10^2}$, were unable to evaluate it correctly. A few candidates did not give their answer in standard form.

(ii) Most candidates rearranged the formula correctly. The most common error was a sign error resulting in an answer of $b = \sqrt{ad + c}$.

(b) Many candidates found this part challenging. The better responses recognised that $m = 3.6$ and proceeded form there. Some complicated solutions were seen.

Question 24

(a) This part was usually correct. A few candidates made an arithmetic error, for example writing $1 - -2 = -3$.

(b) Some candidates found this part demanding and had difficulty multiplying the matrices together. Better responses chose those elements form which an equation in each unknown can be found and then solved.

Question 25

Some good answers were seen to this question. Most candidates were able to correctly find the gradient of *CP* but many then went on to find the equation of the line through *C* (or *P*) with this gradient, and so finding the equation of *CP* and not the equation of the tangent at *P* as requested. Few candidates could find the gradient of the line perpendicular to *CP*.

MATHEMATICS SYLLABUS D

Paper 4024/12
Paper 12

Key messages

To do well in this paper, candidates need to:
- be familiar with all the syllabus content
- be able to carry out basic calculations without a calculator
- produce accurate graphs and diagrams
- set out their work in clear, logical steps.

General comments

In general, candidates were well-prepared for this paper and attempted most of the questions. Candidates across the ability range were able to demonstrate their knowledge, with some straightforward questions that were accessible to all and others offering challenge to the most able.

Many candidates presented their responses well with workings set out legibly and answers clearly stated. Most drew accurate diagrams and graphs using the appropriate geometrical instruments. When a question involves reading values from a graph, such as **Question 12(a)** or **Question 24(b)**, candidates should take care to check the scales, read the required values accurately and show them clearly in their working.

In general, candidates were able to calculate with fractions. They had more difficulty calculating with decimals and standard form, where place value errors were often seen, particularly when dividing by a decimal. Candidates had good basic algebra skills, but many had difficulty manipulating indices and completing the square.

Many candidates would benefit from a greater understanding of the net of a cuboid, matrix manipulation, transformations and histograms. Many candidates found it difficult to access the unfamiliar sequence in **Question 20** and the algebraic fraction problem in **Question 25**.

Comments on specific questions

Question 1

(a) Most candidates were able to convert the decimal to a fraction and write it in its lowest terms.

(b) Most candidates were able to order the fractions correctly, either by using a common denominator or converting to decimals to compare.

Question 2

Many candidates found identifying the symmetries of these shapes challenging. Common errors were to identify 6 lines of symmetry or order of rotational symmetry 6 for the first shape or 1 or 4 lines of symmetry or order of rotational symmetry 1 or 4 for the second shape.

Question 3

Many candidates calculated the two angles correctly as 150° and 90° and drew an accurate, labelled pie chart. Only a minority labelled the sectors just with angles or omitted labels completely. Some candidates drew the sector for Banana as 180° and others drew three equal sectors.

Question 4

(a) Most candidates rounded correctly to the nearest million. The most common errors were truncating to give 64 000 000 or rounding to the nearest ten million to give 60 000 000.

(b) Many candidates are familiar with the requirements of this type of question and clearly show the given calculation with values correctly rounded to 1 significant figure as a first step. Some were unable to square 0.2, with 0.4 commonly seen in place of 0.04. Some of those who correctly found 0.04 were unable to use correct place value when dividing 120 by 0.04 and answers of 300 and 30 were commonly seen. Some candidates did not round all the values to 1 significant figure and others used 7, 5 and 0 in place of 70, 50 and 0.2.

Question 5

(a) Most candidates were able to divide in a ratio and few arithmetic errors were seen. The most common errors were to calculate Jamil's share or to divide by 2 or 7 rather than by 9.

(b) Most candidates were unfamiliar with the process required to increase in a ratio and very few correct answers were seen. The required calculation is $\frac{8}{5}\times 40$ but instead $\frac{5}{8}\times 40$ was often seen. As this calculation leads to a result less than 40, some candidates then added 40 to their result. Some candidates added 40 to the values in the question, used the value of 540 from the previous part, or attempted to divide 40 in the ratio 5:8.

Question 6

The first step required in this question was to use the given volume and base dimensions to calculate the height of the cuboid as 3 cm. Candidates who identified this correctly were often able to draw a correct net. Many candidates did not read the question carefully and drew the net of a 4 by 4 by 4 cube. Some nets were drawn with a missing internal line, with a missing face, or with rectangles positioned incorrectly so they would not fold to make a cuboid. Some candidates attempted a three-dimensional drawing of a cuboid.

Question 7

Many candidates were able to subtract the given probabilities from 1 to reach the correct answer. Some candidates made a place value error, using 0.03 rather than 0.3 in the sum 0.15 + 0.3 + 0.42 to give 0.6 rather than 0.87 which led to the final answer of 0.4. In many cases, however, no working was shown leading to this incorrect answer so the method mark could not be awarded. Some candidates treated this as a number sequence question and attempted to use differences between the given values to find the missing number.

Question 8

Most candidates were able to use a common denominator and subtract the fractions correctly. Incorrect answers were usually a result of an error in converting one of the mixed numbers to an improper fraction or an arithmetic slip when multiplying or subtracting.

Question 9

(a) Most candidates were able to use ruler and compasses to construct the triangle accurately, showing correct construction arcs. The small number of candidates who found the correct position of *C* without including construction arcs were not given full credit.

(b) Many candidates measured the bearing correctly from their scale drawing. The most common errors were to measure angle *ABC* rather than the bearing, to measure the bearing anticlockwise from North or to measure the bearing from the sketch rather than the scale drawing. A small number of candidates gave a length rather than a bearing.

Question 10

(a) Most candidates were able to write 270 correctly as a product of prime factors either using index notation or by writing the product in full as $2 \times 3 \times 3 \times 3 \times 5$, both of which were acceptable. Candidates who showed the correct method of either a factor tree or a factor ladder, but made arithmetic errors when dividing, usually had sufficient correct working to gain a method mark. Some candidates did not complete the prime factorisation, often leaving 9 in their final product.

(b) Many correct answers were seen in this part, although some candidates identified a common factor, or a list of factors (usually 5, 9), rather than the highest common factor of 45. There was some confusion between highest common factor **(HCF)** and lowest common multiple (LCM), and some answers greater than 225 were seen. Often candidates showed correct prime factorisation of 225 even if they did not know how to use this to find the highest common factor.

Question 11

Many candidates were able to use a correct method to solve the simultaneous equations. For this question, the equating coefficients method was often more successful than a substitution method, where errors eliminating fractions sometimes occurred. Candidates who equated the *y* terms sometimes subtracted, rather than added, the two equations. Most solutions were clearly set out, but when candidates identify an error and restart their solution, they should cross out their previous solution to make it clear which is their final answer.

Question 12

(a) Many candidates understood that the average speed is found using total distance divided by total time. Some correctly identified the total distance as 24 km and the total time as 1.5 hours and used these to reach the correct solution. In some cases, incorrect place value adjustments were made when attempting to simplify the division $24 \div 1.5$. Some used 90 minutes in their speed calculation, so did not reach an answer in kilometres per hour, and others used 1.3 rather than 1.5 for 1 hour 30 minutes. A small number of candidates used the time of day, 11 30, rather than the length of time in their calculation. Some candidates did not read the total distance from the graph carefully and 25 was often used in place of 24. Some candidates calculated the area under the graph to find the total distance travelled, which is the method required for a speed–time graph rather than a distance–time graph. Another common misconception was to find the speed for each section of the graph and then find the mean of these three values.

(b) Most candidates interpreted constant speed to be a straight horizontal line and constant deceleration to be a straight line with a negative gradient. Most drew the first section of the speed–time graph correctly to indicate the car travelling at constant speed. It was common to see a line from (80, 10) to (120, 0) for the second section of the graph. This is a result of joining to the end of the time axis rather than using the given deceleration to calculate that it would take a further 20 seconds to come to rest giving an end point of (100, 0).

Question 13

(a) Many correct answers were seen in this part. The most common errors were answers of 5.3×10^5, 53×10^{-6} or 53×10^{-4}.

(b) Many candidates multiplied correctly to reach 12×10^{20}. Some left the answer in this form because they did not recognise that this was not in standard form. Others did attempt to convert to standard form, some correctly, but there were often errors in adjusting the power leading to 1.2×10^{19} rather than 1.2×10^{21}. Some candidates applied index laws incorrectly and subtracted the powers rather than adding them. A small number of candidates attempted to write the given numbers in full and then perform the multiplication; this strategy invariably led to the wrong answer as there were too many zeros in the given numbers.

Question 14

(a) Many candidates were able to give the correct upper bound for the length.

(b) This question required candidates to find the lower bound of the mass of one bag of peanuts and then multiply this by 5 to find the lower bound of 5 bags. Many candidates calculated the mass of 5 bags of mass 80 g and then subtracted 5 g which is an incorrect method to find the lower bound. Those who found the lower bound of one bag as 75 g usually reached the correct answer. A few took 80 g to be given correct to the nearest gram rather than the nearest 10 grams but then used 79.5 g correctly in a calculation for the lower bound of 5 bags.

Question 15

(a) (i) Most candidates drew a correct arc of radius 6 cm meeting the two sides of the quadrilateral as required.

(ii) Candidates who identified that the locus of points equidistant from *QP* and *QR* was the bisector of angle *Q* usually constructed it accurately using correct arcs. Some candidates drew a short bisector that did not meet *RS* and others drew the line *QS*. Some candidates drew perpendicular bisectors of one or more sides and others drew several different arcs with no bisector drawn.

(b) Those candidates who had drawn the correct loci in **part (a)** usually identified the correct region in this part although some shading did not clearly include all parts of the required region.

Question 16

(a) Some candidates were able to apply index laws correctly leading to $-2k = 5$ and the answer $k = -\frac{5}{2}$. In some cases, the negative sign was omitted. Common errors were to add or subtract the powers, rather than multiplying, leading to $k + 2 = 5$ or $k - 2 = 5$ and answers of 3 or 7.

(b) Some candidates cubed the numerator and denominator correctly to give $\frac{x}{8x^3}$ which was sometimes simplified correctly to $\frac{1}{8x^2}$. Common errors were to cube the *x* terms but not the 2, leading to the answer $\frac{1}{2x^2}$, or to cube the numerator but not the denominator, leading to $\frac{x}{2x} = \frac{1}{2}$. Few candidates simplified the terms in the bracket before cubing.

Question 17

Candidates who understood that the given price of \$120 was 75 per cent of the price before the sale usually set up a correct calculation leading to the correct answer of \$160. Some wrote $\frac{75}{100} \times x = 120$ but made errors in the rearrangement. Few candidates recognised this relationship. Common misconceptions were to use \$120 as 25 per cent of the original price, leading to an answer of \$480, or to find 125 per cent of \$120 leading to an answer of \$150.

Question 18

(a) Many candidates started by writing the correct relationship $y = \frac{k}{x^3}$ and substituted $x = \frac{1}{2}$ and $y = 24$ to find the value of *k*. This value was sometimes found correctly as 3 but candidates did not always substitute this back into the original relationship for their final answer, and $y = \frac{k}{x^3}$ was often repeated on the answer line. Many candidates were unable to manipulate the fraction correctly to find the value of *k*. A small number used $y = \frac{k}{x}$, $y = \frac{k}{x^2}$, $y = \frac{k}{\sqrt[3]{x}}$ or $y = kx^3$.

(b) Some candidates who found the correct relationship in **part (a)** reached the correct answer in this part. It was common for candidates to have difficulty in dividing by $\left(\frac{1}{3}\right)^3$ as they did not recognise that this was equivalent to multiplying by 3^3. Those who did identify this relationship often gave the answer 27, as they did not then multiply again by 3, or they made an arithmetic error when evaluating 3×27.

Question 19

(a) (i) Some candidates were able to find the product of the two matrices correctly with few arithmetic errors seen when the correct method was used. Many candidates are unfamiliar with how to carry out matrix multiplication and it was common to see the 2 by 2 matrix $\begin{pmatrix} 100 & 40 \\ 75 & 70 \end{pmatrix}$, the result of multiplying the values in the first column by 2.5 and the second column by 2.

(ii) Some candidates gave a clear explanation that the numbers represented the amount made from ticket sales on Monday and on Tuesday. Many responses were unclear as they also mentioned adults and children: in most cases it was not clear whether these were combined, whether the days were combined, or whether the candidate was referring to four separate amounts. Some candidates did not mention the days in their answer.

(b) Some candidates increased the costs by 10 per cent correctly and gave a correct matrix for their answer. Some misinterpreted the question and increased MN by 10 per cent rather than N by 10 per cent, but could still gain credit for doing this correctly if they had a 2 by 1 matrix as the answer to **part (a)(i)**. As the values are money, answers were expected as decimals, so $\frac{11}{4}$ and $\frac{11}{5}$ were only given partial credit.

Question 20

Many candidates did not realise that they needed to find expressions for the *n*th term of the numerator sequence and the denominator sequence separately and then present these as the numerator and denominator of a fraction for their answer. It was common to see candidates finding differences between the given fractions and go on to use these in an attempt to find a general term, but this approach did not lead to a correct sequence. Those who dealt with the two sequences separately often reached the correct expression for the numerator. They had less success with the denominator sequence, although this was often identified as a quadratic sequence. It was not uncommon to see $n + 5$ as the general term for the numerator sequence rather than $5n + 7$.

Question 21

(a) Most candidates were unfamiliar with writing an expression in completed square form and few correct answers were seen. Some identified that the expression involved $(x+5)^2$ but candidates were less successful at finding the value of *b*: common answers were $(x+5)^2+6$ and $(x+5)^2+31$.

(b) Many candidates did not follow the instruction to use their answer to **part (a)** to solve the equation, so attempts to use the quadratic formula were frequently. When candidates are instructed to use a particular strategy, then this approach is expected. Some candidates who had reached a completed square expression in **part (a)** used it correctly to give a solution here, although many had a positive value for *b* which led to no solutions in this part.

Question 22

Candidates were confident in adding these two algebraic fractions and many correct answers were seen. The most common error was to simplify $3x+15+2x-14$ to $5x-1$ rather than $5x+1$. A small number of candidates added the denominators, rather than multiplying, to find a common denominator. Some cancelled individual terms in the numerator and denominator inappropriately. Those who expanded the product in the denominator usually did it correctly.

Question 23

(a) Many candidates identified that the transformation was a rotation, but few were able to complete the full description correctly. The angle of rotation was usually identified as 90°, but the direction was often given as anticlockwise or omitted. Few candidates were able to identify the centre of rotation correctly as (1, –1). Some candidates identified the transformation as a reflection or a translation or gave more than one transformation in their answer when the question required a single transformation.

(b) Most candidates found using the transformation matrix demanding and few correct answers were seen. Some candidates drew a triangle with two correct vertices, usually (1, 0) and (2, 0), and some showed an attempt to find a product using the given matrix and the position vectors of the vertices of triangle *A*, but errors in the products were common.

Question 24

(a) Some candidates correctly divided the frequency of 12 by the class width of 20 to find the frequency density of 0.6 and drew the correct bar. Many candidates were unfamiliar with how to calculate frequency density and the most common errors were to divide by 10 or 5 rather than by 20. A small number drew multiple bars of different heights rather than a single bar.

(b) Candidates who correctly found the frequencies of the two other groups as 30 and 18 usually calculated the percentage correctly. Many candidates, however, were not able to calculate these frequencies. The most common error was to use a class width of 5 rather than 10 for the $10 < d \leqslant 20$ class, leading to a total of 71 workers rather than 80.

Question 25

This question was found challenging by many candidates. Those who identified that $x^2 - 16$ could be factorised as $(x+4)(x-4)$ were sometimes able to then identify that $(x-4)$ had been cancelled from numerator and denominator to give the simplified expression. This led to an equation for the numerators of $(2x+b)(x-4) = 2x^2 - 5x + a$ which could be expanded and then equating coefficients would lead to the solutions. Often, however, candidates could not identify the step required following identification of $(x+4)(x-4)$. A more complex method was to equate the two fractions and cross multiply to give $(2x^2 - 5x + a)(x+4) = (x^2 - 16)(2x + b)$ which led to a challenging expansion, but if this was done correctly, it would also lead to an identity where the coefficients could be equated leading to the required solutions. It was rare for this approach to lead to the correct solutions due to errors in the expansions.

MATHEMATICS SYLLABUS D

Paper 4024/21
Paper 21

Key messages

Candidates are encouraged to use brackets around negative numbers in working involving powers (**Question 3(a)** and **Question 10(a)(i)**).

Candidates should use a suitable degree of accuracy in their working (**Question 4(b)(ii)**). Final answers should be rounded correct to 3 significant figures where appropriate, or to 1 decimal place for angles, or to the degree of accuracy specified in the question.

It is important to carefully read what is required in a question (**Question 8(a)** and **Question 9(a)(iii)**).

It is also important to carefully read the information given in a question, and use it when forming responses. For example, in **Question 2(c)** and **Question 9(b)** key information was given in the stem of the question.

When asked to explain a given statement, candidates should not repeat what has been stated in the question in their method. They should set their work out logically and clearly, showing all stages of working leading to the given result (**Question 1(c)(i)** and **Question 4(b)(i)**).

General comments

In some questions, candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. Typically, intermediate values should be rounded to at least one more significant figure (or decimal place) than given in the question. It is important that candidates retain sufficient figures in their working and only round their final answer to the required degree of accuracy.

Comments on specific questions

Question 1

(a) The vast majority of candidates answered this part correctly with just a small proportion finding $\frac{22000-7200}{22000}\times100$ instead of the required answer.

(b) The answer to this part was almost always correct.

(c) (i) Care should be taken in a 'show that' question not to use the figure of 1.8 given in the question to prove the result. It is essential to show all working for how K = 1.8 was obtained. Some candidates reached the result K = 0.018 and went on to state 0.018 × 100 so K = 0.018.

(ii) Candidates who were successful on this part used a scaffolded approach, finding the amount in the account after 1 year, 2 years and so on. Many candidates realised that they could use their answer to **part (i)** to construct a response to **part (ii)**.

Question 2

(a) In general, this question was answered correctly by the majority of candidates.

(b) Candidates found this question more demanding than the last part. Data had to be taken from the bar chart and manipulated, and better responses clearly showed all steps in this process.

(c) Candidates were generally successful in this part, working confidently in calculating the mean for grouped data. Note that the number of houses is stated in the stem of the question, as some candidates added the figures from the bar chart to reach 50 and occasionally made an error when doing this.

(d) Most candidates were successful here. Some candidates went on to cancel the fraction of $\frac{10}{50}$, or to give the decimal equivalent, which was not required here.

(e) Candidates found this part challenging. The most common error was not recognising that if two houses are selected, they cannot be the same house, and therefore this is a probability 'without replacement'. Another common error was in not recognising that only one of the houses has exactly 5 residents, therefore one house had 5 residents and the other did not, and that these houses could be chosen in either order.

Question 3

(a) Many candidates performed well on this question. Better responses stated the answer in an equivalent simplified fractional form. As the answer was exact, it should be noted that it should not have been rounded to 3 significant figures, and instead left as 3.125. The main error made was in not writing brackets around –4, and so a small number of candidates incorrectly found $-4^2 = -16$ rather than $(-4)^2 = 16$.

(b) Again, candidates generally were very successful here. A few attempted to convert the expression to an equation and then attempted to find a value for *x* rather than finding an expression in terms of *x*.

(c) Many candidates performed well on this part. The main error was when rearranging $3 - k = 4$. Some candidates attached the negative sign to the 3, rather than the *k*.

(d) Many candidates answered this part correctly. Better responses recognised that the value of *m* could be found by forming a linear equation in *m* from the powers of x. Some made slips when working with the negative numbers.

(e) (i) This part proved difficult for a large proportion of candidates. Note the question asks for an equation to be formed in *x*, not just to find the value of *x*. The more successful attempts used the whole area of the card, 30×24, subtracted the area of the four squares which had been removed, and then equated this result to 576. A common error in setting up the equation was to only set the area of the central rectangle equal to 576.

(ii) Few candidates were successful on this part. The better responses multiplied the three dimensions of the box using their value for *x* and divided this into 1000 to find the simplified fraction of the box filled with sand.

Question 4

(a) (i) The vast majority of candidates performed well on this question, with clear intersecting arcs shown. Occasionally the 50° angle was incorrectly measured.

(ii) Just over half of candidates could successfully measure the angle *ADC* from their construction.

(iii) Candidates tended to be more successful measuring the length of the line and finding the perimeter than measuring the angle in **part (a)(ii)**. Occasionally the distance *AC* was incorrectly added for the perimeter.

(b) (i) Candidates could attempt this question either with two stages of two-dimensional application of Pythagoras' theorem, or a one-stage three-dimensional application of Pythagoras' theorem. Better responses from the first approach used exact, and not rounded, values in the second stage. As this is a 'show that' question, it was essential for candidates to present all steps in their working to reach their answer, and to give the numerical answer to a greater degree of accuracy than that already stated in the question.

(ii) The most direct route to find the required angle was to find *TX* and *RX* using Pythagoras' theorem, then the cosine rule to find angle *TRX*. Again, candidates should be careful to use exact and not rounded values in this multi-stage question.

Question 5

(a) Many candidates found this part challenging.

(b) (i) Many candidates were successful with this part, applying their understanding of rotational and line symmetry in conjunction with Venn diagrams.

(ii) Many candidates could correctly list the elements in the set.

(iii) Few candidates were successful on this part. Some listed the elements rather than writing the number of elements in the set.

(iv) Few candidates were successful on this part although some novel responses were seen.

Question 6

(a) The answer to this part was almost always correct.

(b) Where a candidate understood to begin with $f(x) = 2$ they invariably went on to find a correct solution. The most common error here was to calculate f(2) rather than to set $f(x) = 2$ and rearrange to solve.

(c) Most candidates were successful here, having been well-prepared in finding an inverse function. There were occasional sign errors.

(d) Many candidates had difficulty establishing the first step towards solving this problem. Few candidates reached the step of forming the equation $f(x) = g(x) + 4$ or equivalent, with many often multiplying one of the equations by 4 or by adding 4 on to the wrong side of the equation.

Question 7

(a) The key to being successful here was in recognising the first graph had a positive gradient and a negative *y*–intercept, with the opposite being true for the second graph. More successful responses clearly identified which elements of a straight line graph equation represent the gradient and which the *y*–intercept.

(b) Various approaches worked well here, but the most successful was that which identified –3 and 2 as roots of the quadratic and hence that $(x + 3)$ and $(x – 2)$ were factors. Expansion and simplification of $(x + 3)(x – 2)$ led to correct values of *a* and *b*. Some candidates who substituted the values of *x* into the given general equation of the curve were also successful, but many did not also substitute $y = 0$, or had problems when trying to square –3.

(c) (i) Few candidates presented a method which identified that, for the equation to have two solutions, a horizontal line should intersect the graph at only two points. Better responses clearly identified the required values of *k* which matched the values of *y* where the two stationary points were situated, at –4.1 and 8.2.

(ii) As the question states the method to be used here, this approach was expected from candidates. Better responses showed a suitable line drawn on the grid, formed from a comparison of the cubic function given in this part with that of the cubic graph given on the grid. By comparing the two, the equation was then solved by finding where the curved graph already drawn and the straight line graph $y = 2x + 1$ intersected.

Question 8

(a) Better responses considered the sum of the angles in triangle *PQR* and that angles *OPQ* and *OQP* had to be equal, as triangle *OPQ* is isosceles. Some candidates gained credit by writing on the diagram that angle *PRQ* = 4*x* but did not make further progress. A common error here was to assume that angle *OQR* = 90°. Some candidates found the value of *x* which was not required here.

(b) There were two equally successful approaches used here. The first was to use similar triangles to work out the length of *JK* which is 17.5. Using equal opposite angles, the required angle *y* was then found using the given area and the triangle area formula $\frac{1}{2}ab\sin C$ on triangle *JKN*. The second approach was to use the given area and an area scale factor to find the area of triangle *KLM* which is 27. Once again, the triangle area formula $\frac{1}{2}ab\sin C$, along with the known sides, was used on triangle *KLM* to find the required angle *y*. A common error was to attempt the problem without use of the area of triangle *JKN*.

Question 9

(a) (i) The answer to this part was almost always correct.

(ii) Few candidates used the correct method, which recognised that if 48 candidates passed the test then 32 candidates failed, and to then read at a cumulative frequency of 32 to find the pass mark of 44. Many candidates found 60 per cent of 80 (48) and read at this cumulative frequency value to get a mark of 52. Candidates should note this figure of 52 is gained by the bottom 60 per cent of candidates, not the top 60 per cent.

(iii) The common error here by many candidates was to find the cumulative frequency rather than the frequency.

(b) Candidates could often successfully find the mid-interval values and set up an equation for calculating the mean from grouped data. The better responses went on to set up a second simultaneous equation in *p* and *q* and then solving the two equations to find *p* and *q*. Some calculated the number of students completing the test; information which was provided in the stem of the question. Some candidates divided their total amount of minutes by (8 + 13 + *p* + 20 + *q*), without recognising that this total frequency is 80.

Question 10

(a) (i) Some candidates understood the approach required in this part, but made slips in squaring –3 or did not use brackets around the negative number. There were some candidates who did not recognise the notation for magnitude of the line *AB* and therefore did not know Pythagoras' theorem was required here.

(ii)(a) Candidates found this part challenging, often reversing the signs in their answer. Some candidates sketched a simple diagram to support their understanding; this is encouraged if candidates find it helps them visualise the problem.

(ii)(b) Some candidates again drew a simple diagram, from which they could show that the vector in the stem could also be used to take them from *B* to *D*, as *B* is the midpoint of *AD*.

(b) (i) Approximately half of candidates were successful here. A common wrong answer seen was **p – q** rather than the correct answer of **q – p**.

(ii) Many candidates found a correct vector route for $\overrightarrow{OS}$ along the lines in the diagram. The most successful approach considered the route $\overrightarrow{OP}+\overrightarrow{PS}$, with $\overrightarrow{PS}$ equal to half of $\overrightarrow{PQ}$ already found in **(b)(i)**.

(iii) Many found a correct vector route for $\overrightarrow{SR}$ along the lines in the diagram. Better responses recognised the most efficient route to take is $\overrightarrow{SO}+\overrightarrow{OR}$, where R is $\frac{2}{3}$ of the way along $\overrightarrow{OQ}$ and where $\overrightarrow{OS}$ has already been obtained in **(b)(ii)**.

MATHEMATICS SYLLABUS D

Paper 4024/22
Paper 22

Key messages

Candidates are expected to understand the need to work to accuracies of more than 3 significant figures, or more than one decimal place for angles, when calculations involve a number of steps in the working; only then can the final answer be given to an appropriate level of accuracy. This was not always followed. This was particularly evident on **Question 7(c)** with candidates working with the angles to one decimal place which resulted in an inaccurate value for the required angle.

Candidates need to consider whether the values they are using or answers they are giving are sensible values. For example, **Question 1(c)** resulted in several candidates saying that the difference in amount of money in the accounts was $3494.51 … nearly as much as was invested. Also, on **Question 9(b)**, several candidates used the height of the cone as 10 cm, which is larger than the slant height of the cone.

General comments

Overall, the presentation of candidate's work was good, with clear and legible working. All candidates made some attempt at the majority of questions on the paper. A continued improvement was seen in the angle question where reasons were required, but few candidates clearly indicate which angle they are referring to when using three letter notation.

Comments on specific questions

Question 1

(a) The majority of candidates correctly found the sale price, with many choosing to find 15 per cent and then use this value to calculate the sale price. The most common error when using this approach was to add $11.70 to the price of the drill.

(b) Nearly all candidates were able to deal correctly with the exchange rates and find the amount of money in dollars after the purchase. Most chose to convert the money to dollars, subtract the price of the clock and then convert back to dollars, with a lot fewer choosing to convert the cost of the clock into dollars.

(c) Many candidates obtained full marks on this question. Some had the correct method for both accounts but the accuracy they used resulted in a difference of $5.50, while others quoted the higher value rather than the difference. A common error was to compare the amount in the account with compound interest with the amount of simple interest earned. A common wrong answer was $14 from calculating both cases as simple interest. Some candidates took the amount they obtained using the compound interest formula to be the interest rather than the amount in the account and so added $3500 to this answer. Most candidates who obtained a difference in interest of around $3500 would have benefitted from considering the appropriateness of their answer in the context of the question.

Question 2

(a) Most candidates plotted the five points accurately.

(b) Many correct answers were seen. Common incorrect answers usually involved a comment about the variables either being negatively correlated or inversely proportional and to a lesser extent that they both increased or decreased together.

(c) The candidates were slightly less successful in this part. A small majority drew an acceptable line of best fit and were able to give a correct estimate of the number of cups of hot chocolate. Some earned partial credit for either an acceptable line of best fit or for a correct reading from an incorrect line. Incorrect lines usually resulted from an incorrect gradient and occasionally were too short to cover an adequate range of points. An incorrect reading usually involved giving an answer that was not an integer value; an integer value is expected as this is appropriate in the context of the question.

Question 3

(a) The majority of candidates answered this correctly. Mistakes that were made usually resulted from confusion over the minus signs, for example $4a - 7a - b + 6b = -3a - 7b$.

(b) This was often answered correctly. Common errors included multiplying by 2 incorrectly and obtaining either $m - 4 = 10$, or $m - 8 = 5$, or obtaining an answer of $\frac{9}{2}$ from $\frac{m}{2} = 9$.

(c) Most candidates were able to obtain a correct expression for t. The most common approach was to correctly multiply both sides by 3, but errors were sometimes seen by candidates who incorrectly added 4 or divided by 4 in order to have the 4 on the other side of the equation.

(d) Candidates again demonstrated a good understanding of the algebra needed to expand this bracket. The most common errors were answers of $6y^2 + 15y$ or $6y^3 + 15$. Occasionally, having obtained the correct expansion, candidates would introduce errors by attempting further work.

Question 4

(a) Most candidates were able to answer this part of the question correctly than the rest of the parts. The most common error was to state that the mode is 28.

(b) Most candidates correctly found the mean. One common error was to divide the total number of emails by 8 while another was to sum the frequencies and divide this by 8.

(c) Again, most of the candidates gave the correct fraction. Some candidates gave an equivalent fraction or decimal. There were candidates who gave the correct probability for the number of candidates who received 4 or fewer emails that day.

(d) Many candidates were able to estimate the number of adults who received exactly 5 emails. A common mistake was to use the number 5 in their calculation, e.g. $\frac{18000}{5}$ or $\frac{5}{15} \times 18000$. It was not uncommon for candidates to state the number of adults in the town as 1800 and use that number in their calculation.

Question 5

(a) Many candidates described the set correctly, with both $(A \cup B)'$ and $A' \cap B'$ seen. A common error was to write $A' \cup B'$, or give answers containing the symbols $\subset$ or $\subseteq$.

(b) (i) All the numbers were placed correctly by most candidates. Of the remaining candidates, most were able to gain some credit by demonstrating an understanding of where to place the majority of the numbers. Errors including the wrong positioning of 3 and 6, omitting 7, 10 and 11, repeating some elements and occasionally including the numbers 1, 18 and/or 36.

(ii) Few candidates were able to list the elements in the given subset.

(iii) The notation $\mathrm{n}(P \cup Q)$ appeared to be unfamiliar to a large number of candidates with many listing the elements and others giving an answer involving *P*, *Q* and/or *R*.

(iv) Very few correct answers were seen in this part. Some attempted to describe a set using set notation, however the subset given was not an empty set. About a quarter of the candidates did not attempt to answer this part of the question.

Question 6

(a) Many candidates were able to interpret all the information and correctly gave the coordinates for one position of *R*; the most common ones being (2, 2), (6, 6) and (7, 7). Occasionally the coordinates (1, 1) or (5, 5) were stated which produced a right-angled triangle. It was not uncommon to see the coordinates (3, 3) which produced a straight line.

(b) Many candidates were able to state the correct equations for all three curves. Some candidates obtained some correct equations, with few understanding the basic shape of both quadratic and cubic curves.

(c) Some candidates were able to obtain the correct gradient for the perpendicular but then found the equation of the line with this gradient through either *A* or *B*. Some obtained the gradient of the line *AB* and then used this gradient to find the equation of *AB*, using either *A* or *B* or the midpoint of *AB*. Some used an incorrect method for the gradient of *AB* but then used this value to find the gradient of a perpendicular.

Question 7

(a) (i) The majority of candidates knew how to find the perimeter of a rectangle. The most common wrong answer was 75.

(ii) Many candidates were able to find the length of the diagonal correctly using Pythagoras' Theorem. Wrong answers included the value 75, from adding the sides of the rectangle rather than calculating the length of the diagonal.

(b) Many candidates were able to set up a correct equation and solve this to find the value for *a*. Some candidates who set up the equation correctly made mistakes, commonly with $(a + 4)^2$ being expanded to $a^2 + 16$, or $2 \times a \times 11 \times \cos 60$ being simplified incorrectly by being separated into two terms. Those who chose to set the equation up by writing $\cos 60 = ….$, were more likely to make mistakes when rearranging to find the value of *a*. Mistakes were also seen with candidates not linking the sides correctly in the cosine formula, as well as several candidates incorrectly setting up an equation by using the sine rule.

(c) Many candidates were able to obtain a correct value for angle *DAB*. Some candidates did not obtain an acceptable value for the angle as premature approximation was used when calculating angles *DAC* and *CAB*. Accuracy errors were also made when candidates chose to calculate angle *DAC* via the length *AD*. Some candidates only looked at triangle *DAC* and gave angle *DAC* as the required angle. Some candidates assumed that triangle *ABC* was right-angled and used this to calculate angle *CAB* by using the 8 and 9. Others found the area of triangle *ACB* and found the height of the triangle and assumed this was *BC*. There were candidates who were able to use the area of the quadrilateral correctly but did not make further progress.

Question 8

(a) The correct answer was given by the majority of candidates.

(b) Candidates found this part of the question challenging and few correct answers were seen. Some were able to set up the inequality and find the correct inequality for *x*. Not all were able to state the correct largest integer satisfying that inequality but instead gave an incorrect integer whilst others just stated the inequality as the answer. A common error was to multiply both the numerator and denominator by 3 to obtain an expression for $3\mathrm{g}(x)$. Another error seen was to obtain the inequality $x > –9$ from the inequality $– x > 9$.

(c) More candidates were able to score full marks on this part of the question than the previous part. However, a significant number were not able to set up the equation in *x* correctly and so were unable to make further progress. From those candidates who did set up a correct equation, mistakes were seen in both the expansion of brackets in the numerator and eliminating the fraction.

(d) Nearly half the candidates obtained the correct answer here, many by finding the inverse of g(x) and then setting this equal to 5 rather than calculating g(5).

Question 9

(a) A minority of candidates scored full marks on this question. Those candidates who were able to set up the correct equation occasionally lost accuracy by choosing to use a value of pi rather than divide the whole equation by pi. The most common incorrect attempt was made by those who equated the given total surface area of the cone to the curved surface area, $6\pi l$, omitting the area of the base. Some candidates chose to substitute the value of 8 into the formula for the surface area rather than showing that the value of l is 8.

(b) A minority of candidates were able to work out the volume of the cone. A common error was to use the slant height instead of the vertical height in the formula. A few candidates recognised the need to use Pythagoras' theorem to calculate the vertical height but incorrectly added 8^2 and 6^2.

(c) The correct radius of the similar cone was found by few candidates. Most candidates used one of the following two incorrect methods; they either set up an equation for the total surface area of the smaller cone but used its slant height as 8 cm, or they wrote the ratio of the radii equal to the ratio of the surface areas. Few candidates recognised the need to find the square root of the ratio of the areas to evaluate the radius of the smaller cone.

Question 10

(a) Few candidates stated the correct value of angle *ACB* giving complete reasons. Often candidates did not make it clear which angles they were referring to while others did not give reasons for each angle they found. Occasionally candidates arrived at the correct value of 40° for the angle but assumed that angle *CAB* was 90° and angle *CBA* was 50°.

(b) (i) Several candidates were able to appreciate the need to divide the quadrilateral into two congruent right-angled triangles to find the length of *PR*. Having done this, a correct trigonometric formula was normally used, however some incorrectly found the length of *OR*.

(ii) Of those candidates who made some attempt at this part of the question, many were able to find the area of the sector and also had the correct method for the area of the quadrilateral using their value of *PR*. Some candidates calculated the percentage correctly. A common mistake was to find the shaded area as a percentage of the area of the quadrilateral added to the area of the sector.

Question 11

(a) Completing the tree diagram proved challenging for many candidates and fully correct responses were in the minority. Some candidates seemed unfamiliar with giving probabilities as algebraic expressions, giving instead numerical values. Having been given the denominator of 11 on one of the branches for the second ball, some candidates used a denominator other than 11 for the rest of these probabilities.

(b) This part proved more demanding than the previous part. Some candidates worked from their tree diagram to show a correct method for at least one of the relevant branches. Some candidates added the pairs of probabilities together rather than multiplying them. Other candidates used either the wrong branches or probabilities that were not on the tree diagram.

(c) Few fully correct solutions were seen. Some were able to set up a correct product and equate it to the given probability but lacked the necessary algebraic skills to continue to a correct solution. A large proportion of candidates were unable to set up a correct equation, either equating the given probability to just one probability (not always from their tree diagram) or using probabilities from an incorrect branch.

Question 12

(a) (i) Correct answers were in the minority. Some candidates were able to get one component correct. However, a common wrong answer was $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, from $\overrightarrow{OA} + \overrightarrow{OB}$.

(ii) Few candidates were able to find the correct coordinates of *C*. A common wrong answer was (–7, 8).

(iii) Very few candidates found the solutions of *n*. Some were able to set up a correct equation using Pythagoras' theorem but often made algebraic mistakes when attempting to solve this. Others incorrectly set up an equation using Pythagoras' theorem by adding the coordinates.

(b) Many candidates found this vector question challenging. Most success was had by candidates who started with a vector route for $\overrightarrow{KL}$, however occasionally there were errors seen in some attempts. Some candidates did not realise that a numerical ratio was needed and used vectors in their answer. A correct vector for $\overrightarrow{PL}$ or $\overrightarrow{LR}$ was sometimes seen but these candidates were then unable to obtain the required ratio. The most common wrong answer was 1:2, not always supported by working producing that ratio.